Chapter 10

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Rotation Of A Rigid Object About A Fixed Axis

1

Lecture 03

2

- Rotational Kinetic Energy
- Moment Of Inertia

2

Rotational Kinetic Energy

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An object rotating about some axis with an angular speed, ω, has rotational kinetic energy even though it may not have any translational kinetic energy.

Each particle has a kinetic energy of

$$K = \frac{1}{2} m_i v_i^2$$

Since the tangential velocity depends on the distance, r, from the axis of rotation, we can substitute

$$v_i = r\omega_i$$

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Rotational Kinetic Energy, cont

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- The total rotational kinetic energy of the rigid object is the sum of the energies of all its particles.
- $K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i r_i^2 \omega^2 = \frac{1}{2} \omega^2 \sum_i m_i r_i^2$

$$K_R = \frac{1}{2} I \omega^2$$

- I is called the moment of inertia.

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Rotational Kinetic Energy, final

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- There is an analogy between the kinetic energies associated with linear motion ($K = \frac{1}{2} m_i v_i^2$) and the kinetic energy associated with rotational motion ($K_R = \frac{1}{2} I\omega^2$).
- Rotational kinetic energy is not a new type of energy, the form is different because it is applied to a rotating object.
- The units of rotational kinetic energy are Joules (J).

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Moment of Inertia

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The definition of moment of inertia is

$$I = \sum_i m_i r_i^2$$

- The SI units are: $kg.m^2$
- Mass is an inherent property of an object, but the moment of inertia depends on the choice of rotational axis.
- Moment of inertia is a measure of the resistance of an object to changes in its rotational motion.
- The moment of inertia depends on the mass and how the mass is distributed around the rotational axis.

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Moment of Inertia

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- The moment of inertia of a system of discrete particles can be calculated by applying the definition for I.
- For a continuous rigid object, imagine the object to be divided into many small elements, each having a mass of Δm_i.
- We can rewrite the expression for I in terms of Δm.

$$I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i = \int r^2 dm$$

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Parallel-Axis Theorem

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In the previous examples, the axis of rotation coincided with the axis of symmetry of the object.

For an arbitrary axis, the parallel-axis theorem often simplifies calculations.

The theorem states

$$I = I_{com} + MD^2$$

- I is about any axis parallel to the axis through the center of mass of the object.
- I_{com} is about the axis through the center of mass.
- D is the distance from the center of mass axis to the arbitrary axis.

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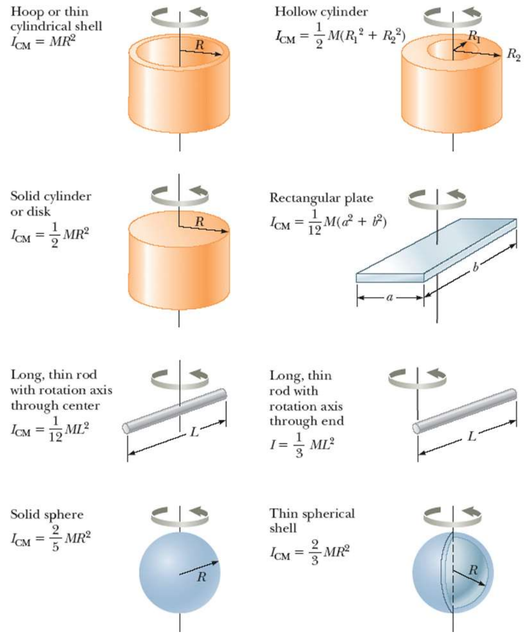

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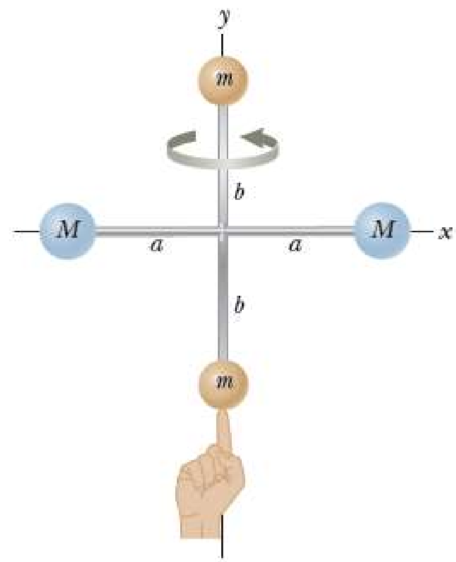
An Unusual Baton

Saturday, 30 January, 2021 16:18

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Four tiny spheres are fastened to the ends of two rods of negligible mass lying in the xy plane to form an unusual baton. If the system rotates about the y axis with an angular speed ω, find the moment of inertia about this axis.



$$I_y = \sum_i m_i r_i^2 = Ma^2 + Ma^2 = \boxed{2Ma^2}$$

$$K_R = \tfrac{1}{2} I_y \omega^2 = \tfrac{1}{2}(2Ma^2)\omega^2 = \boxed{Ma^2\omega^2}$$

Applying the Parallel-Axis Theorem

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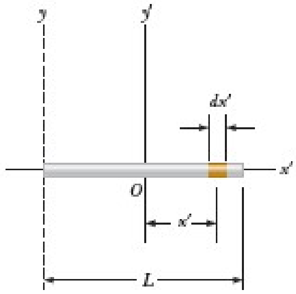

Find the moment of inertia of the rod about an axis perpendicular to the rod through one end (the y axis in the figure).

$$I = I_{\text{CM}} + MD^2 = \tfrac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \boxed{\tfrac{1}{3}ML^2}$$

An Unusual Baton

Saturday, 30 January, 2021 16:19

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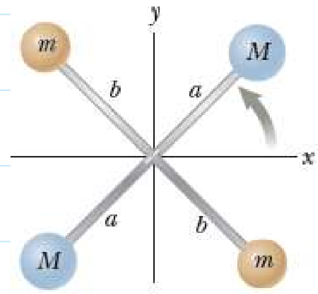
- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Four tiny spheres are fastened to the ends of two rods of negligible mass lying in the xy plane to form an unusual baton. Suppose the system rotates in the xy plane about the z axis through the center of the baton. Calculate the moment of inertia and rotational kinetic energy about this axis.



$$I_z = \sum_i m_i r_i^2 = Ma^2 + Ma^2 + mb^2 + mb^2 = 2Ma^2 + 2mb^2$$

$$K_R = \tfrac{1}{2} I_z \omega^2 = \tfrac{1}{2}(2Ma^2 + 2mb^2)\omega^2 = (Ma^2 + mb^2)\omega^2$$